2D and 3D Vector Mathematics

Why Vectors, Why not Points?

- maths notes Chapter 3 . . .
- vector maths is easy — *linear algebra*
- most vector transformations implementable by matrix multiplication
- . . . except *translation* (shift)
- a vector has just *magnitude* and *direction*

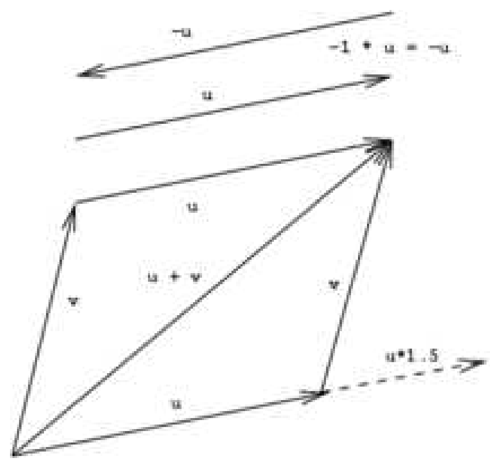

Figure 1: Vectors in 2D.

- although a vector has no position . . .
- vectors represent *displacement*, when used in computer graphics, they represent displacement from the *origin* (a *point*)
- $point + vector = point$

Vector Algebra

- Addition. In Figure 1 this is done using either (i) the *head-to-tail triangle rule* or (ii) the *parallelogram* rule; they are equivalent and give the same result. There is a zero vector $\mathbf{0}$ for which $\mathbf{u} + \mathbf{0} = \mathbf{u}$.
- Multiplication by a scalar, (*scaling* the vector). For example, in Figure 1 see the effect of multiplying vector $\mathbf{u}$ by 1.5; the direction stays the same, magnitude is multiplied by 1.5.

Vector Basis

- we represent a vector as a sum of *basis* components
- usually the basis is *orthonormal* . . . the basis vectors are *orthogonal* and *normalised* = *unit length*

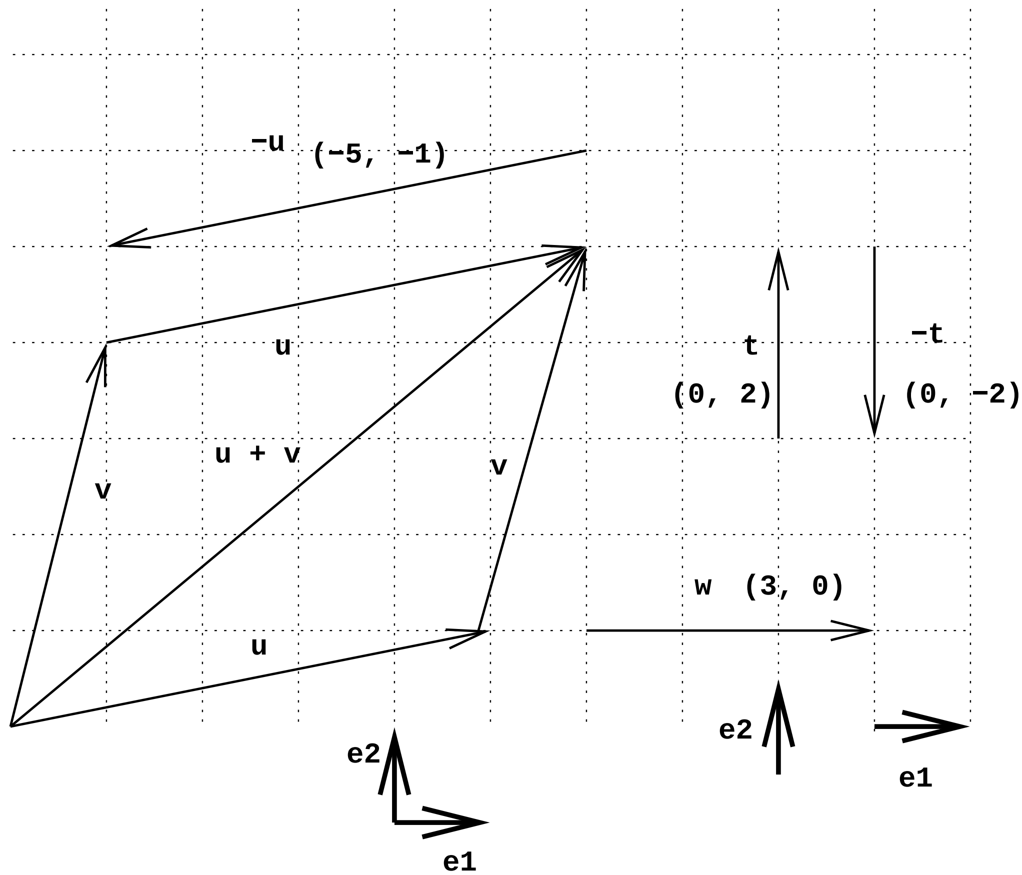

Figure 2: Vector Basis.

- can now do our basic vector operations by working on the individual components

  (a) Add two vectors; $\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_y + v_y) = (5 + 1, 1 + 4) = (6, 5)$, which can be verified by examining Figure 2;

  (b) Multiply by a scalar; multiply each component by the scalar; example in Figure 2, $-1\mathbf{u} = -\mathbf{u} = (-5, -1)$;

  (c) Zero vector, $\mathbf{0} = (0, 0)$.

Scalar (dot) product

- The scalar product is defined as

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta, \tag{1}$$

where $\| \ \|$ denotes *magnitude* and θ is the angle between $\mathbf{u}$ and $\mathbf{v}$

- if basis is orthogonal then easier formula

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2. \tag{2}$$

-

$$\theta = \cos^{-1}(\mathbf{u} \cdot \mathbf{v} / \|\mathbf{u}\| \|\mathbf{v}\|). \tag{3}$$

- *direction cosines*
- components of a *unit vector* are its *direction cosines*, i.e. the *cosines* of the angles it makes with the $\mathbf{e_1}$ and $\mathbf{e_2}$, respectively

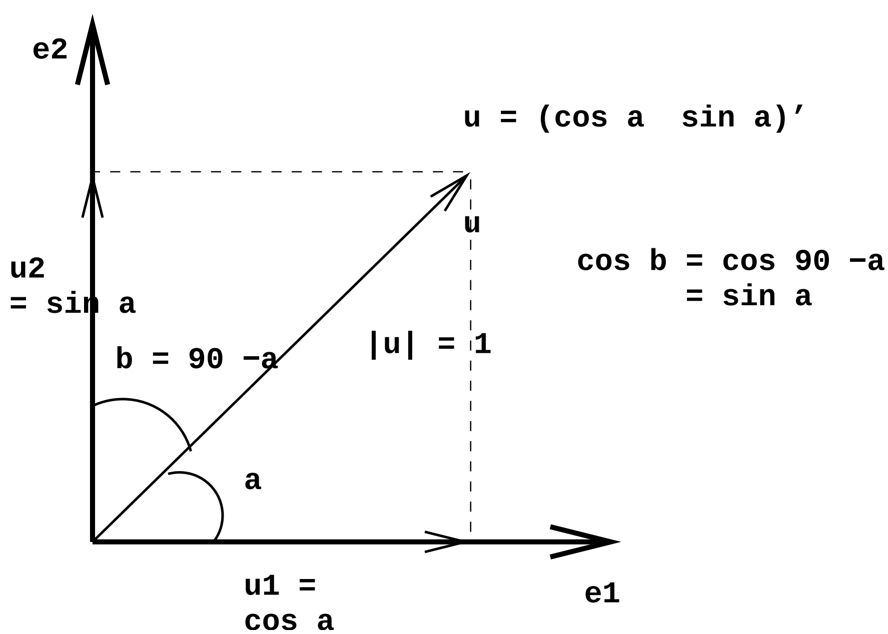

Figure 3: Direction cosines.